JOINT UNIVERSITIES PRELIMINARY EXAMINATIONS BOARD

JUNE 2019 EXAMINATIONS

JUPEB/015

MATHEMATICS: SCI – J154 **Time Allowed: 3 Hours**

SECTION A: MULTIPLE CHOICE QUESTIONS

Answer all questions in this section.

Use the OMR answer sheet provided to answer the questions. Follow the instructions on the OMR sheet.

SECTION B: ESSAY QUESTIONS

Answer FOUR questions; ONE question from each course.

SECTION A: MULTIPLE CHOICE QUESTIONS.

Answer ALL Questions

1. Let $Q, \mathrm{N}, \mathrm{Z}, C$ and Q^{c} denote the set of Rational, Natural, Integer, Complex and Irrational numbers respectively. Which of the following set contains the whole set of real numbers?
   A. $\mathrm{N} \cup \mathrm{Z}$
   B. $\mathrm{Z} \cup C$
   C. $Q \cup Q^{c} \cup \{0\}$
   D. $Q \cup C$

2. Determine the solution set of the inequality: $2x^{2}-3x-2<0$
   A. $(2,\infty)$
   B. $\left(-\infty,-\frac{1}{2}\right)$
   C. $\left(-\frac{1}{2},2\right)$
   D. $\left[-\frac{1}{2},2\right]$

3. Evaluate the conjugate of the complex number: $z=\frac{1}{-1+i}$.
   A. $\frac{1}{-1-i}$
   B. $\frac{-1+i}{2}$
   C. $-1-i$
   D. $-1+i$

4. Compute the value of x if $\frac{4^{-x}\times 32^{x+1}+5x}{5}=\frac{16^{2x+1}}{5}+x$
   A. $1/5$
   B. $2/5$
   C. $3/5$
   D. $4/5$

5. Find $\tan 75^{\circ}$ in surd form.
   A. $\frac{(\sqrt{3}-2)}{2}$
   B. $\frac{(\sqrt{3}+2)}{2}$

C. $\frac{(2-\sqrt{3})}{2}$

D. $\frac{(\sqrt{2}-3)}{2}$

6. Given that $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$ and $\log_{10} 7 = 0.8451$, compute $\log_{14} 72$.

A. 0.6502
B. 1.6205
C. 3.6105
D. 1.5025

7. If $U_1 = 3, U_2 = 4$ and $U_n = a + bn$, compute U_6.

A. 3
B. 4
C. 2
D. 8

8. Let α and β be the roots of the equation: $x^2 + 5x + 6 = 0$. Deduce the equation whose roots are α^2 and β^2.

A. $x^2 - 13x + 36 = 0$
B. $x^2 - 5x + 13 = 0$
C. $x^2 + 13x + 36 = 0$
D. $x^2 - 13x - 13 = 0$

9. Which form of partial fraction is equivalent to the algebraic fraction: $\frac{f(x)}{(x^2+1)(x-b)^2}$

A. $\frac{Ax+B}{x^2+a} + \frac{Cx+D}{(x-b)^2}$
B. $\frac{Ax+B}{x^2+a} + \frac{C}{x-b} + \frac{D}{(x-b)^2}$
C. $\frac{Ax+B}{x^2+a} + \frac{Cx+D}{x-b} + \frac{Ex+F}{(x-b)^2}$
D. $\frac{Ax+B}{x^2+a} + \frac{C}{x-b} + \frac{D}{(x-b)^2}$

10. Determine the centre and the radius of the circle $x^2 + y^2 - 2ax + 2yb + a^2 = 0$ in terms of a and b.

A. Centre $(-a, -b), radius = b$
B. Centre $(a, b), radius = a$
C. Centre $(a, -b), radius = b$
D. Centre $(-a, b), radius = a$

11. Express $1 + i$ in the conjugate of its polar.

A. $\sqrt{2}\,(\cos 45^0 + isin45^0)$

B. $\sqrt{2}\,(\cos 315^0 + isin315^0)$

C. $\sqrt{2}\,(\cos 315^0 - isin315^0)$

D. $2(\cos 45^0 - isin45^0)$

12. A binary operation, '$*$' is defined on the set of real numbers, $\boldsymbol{R}$ by: $x * y = \frac{1}{5}(5x + 5y - 4)$ for any two real numbers $x, y \in \boldsymbol{R}$. Calculate the identity element.

A. 5

B. 4

C. $^5/_4$

D. $^4/_5$

13. If $P = \begin{bmatrix} y & 3 & 0 \\ 2 & 1 & y \\ 2 & 1 & 2 \end{bmatrix}$, what is the range of values of y for which $|P| > 0$?

A. $-2 < y < 6$

B. $2 < y < 6$

C. $-6 < y < 2$

D. $-6 < y < -2$

14. Calculate the sum to infinity of $1 + \tan 30° + \tan^2 30° + \tan^3 30° + \cdots$

A. $\frac{1}{2}(3 + \sqrt{3})$

B. $\frac{1}{3}(2 + \sqrt{3})$

C. $\frac{1}{2}(3 - \sqrt{3})$

D. $\frac{1}{2}(2 - \sqrt{3})$

15. Estimate the value of the product $\log_2 3 \log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8$.

A. 1

B. 2

C. 3

D. 4

16. Let the expression $ax^2 + bx + c = 8$ at $x = 1$. If the derivative is $4x + 2$, determine the values of a, b and c respectively.

A. 2, 4, 2

B. 4, 4, 0

C. 2, 2, 4

D. 4, 2, 2

17. The distance, S in metres moved by a particle in time, t in seconds is given by $S(t) = 3.5t^3 - 2t$. Evaluate its speed after 2 seconds.

A. $22\ ms^{-1}$

B. $40\ ms^{-1}$
C. $24\ ms^{-1}$
D. $28\ ms^{-1}$

18. Calculate the gradient of the curve $y = x^3 - 3x^2 + 7x + 23$ at the point (2, 1).
A. 7
B. 38
C. 4
D. 28

19. Find the derivative of the line joining the points $A(3, 5)$ to $B(6, -8)$.
A. $-\frac{13}{3}$
B. $\frac{13}{3}$
C. 22
D. -22

20. Evaluate: $\lim_{x \to 0}\left(\frac{x + Sinx}{x(x+1)}\right)$.
A. 0
B. -2
C. 2
D. ∞

21. Given that $x = sin(t)$ and $y = cos(t)$, then the letter t is simply called the _____
A. Time.
B. Variation.
C. Parameter.
D. Differential value.

22. Given that $y = \cos 2\theta$, $x = \sin\theta$. Express $\frac{dy}{dx}$ in terms of θ.
A. $-4\sin\theta$
B. $-2\sin 2\theta$
C. $4\cos\theta$
D. $-4\cos\theta\sin\theta$

23. Let the given curve of a parabola be $y^2 = x$. Estimate the area of the curve in terms of x.
A. $\frac{x}{2} + c$
B. $\frac{2x}{7} + c$

C. $\frac{2x^{3/2}}{3} + c$

D. $2x + c$

24. Find the derivative of the function $y = \log_e \tan 2x$.

A. $2sec^2 2x$

B. $\frac{2}{\tan 2x}$

C. $\frac{2sec^2 2x}{\tan 2x}$

D. $\frac{sec^2 2x}{\tan 2x}$

25. The radius of a sphere is increasing at the rate of 0.1cm/s. Determine the rate at which the surface area is increasing when the radius is 5cm.

A. $8\pi cm^2/s$

B. $6\pi cm^2/s$

C. $5\pi cm^2/s$

D. $4\pi cm^2/s$

26. Evaluate y in terms of x if $\frac{dy}{dx} = 3x^2 - 6x + 2$, given that $y = 7$ when $x = 0$.

A $y = x^3 - 3x^2 + 2x$

B $y = x^3 - 3x^2 + 5x + 3$

C $y = x^3 - 3x^2 + 2x + 7$

D $y = x^3 - 3x^2 + 2x + 5$

27. Obtain $\frac{d^2y}{dx^2}$ for the function: $y = 2x^{2/3} - 5x^{1/2}$.

A. $\frac{-2}{9}x^{-\frac{2}{3}} - \frac{5}{4}x^{-\frac{3}{7}}$

B. $\frac{-4}{9}x^{-4/3} + \frac{5}{4}x^{-3/2}$

C. $\frac{-4}{9}x^{-4/9} + \frac{5}{4}x^{-3/7}$

D. $\frac{-4}{3}x^{-\frac{4}{3}} - \frac{1}{4}x^{-\frac{3}{2}}$

28. Evaluate $\int_o^z 12x^2 dx = 1372$ to get the value of z.

A. -14

B. 14

C. 7

D. 8

29. The expression for e^{-x} is given as …

A. $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots$

B. $1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\ldots$

C. $1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots$

D. $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}-\ldots$

30. Describe the unit vector in the direction of $2\boldsymbol{i}+\boldsymbol{j}-2\boldsymbol{k}$.

A. $\frac{2}{3}\boldsymbol{i}+\frac{1}{3}\boldsymbol{j}-\frac{2}{3}\boldsymbol{k}$

B. $\frac{2}{5}\boldsymbol{i}+\frac{1}{5}\boldsymbol{j}-\frac{2}{5}\boldsymbol{k}$

C. $\frac{2}{7}\boldsymbol{i}+\frac{1}{7}\boldsymbol{j}-\frac{2}{7}\boldsymbol{k}$

D. $\frac{2}{9}i+\frac{1}{9}j-\frac{2}{9}k$

31. Compute the modulus of the vector $2\boldsymbol{i}+3\boldsymbol{j}-\sqrt{3}\boldsymbol{k}$.

A. 2

B. 3

C. 4

D. 5

32. An object of mass 2.5 kg moving at a constant velocity of $12ms^{-1}$ comes in contact with a resistance of 10 N. How long does it take the body to come to rest?

A. $2\,s$

B. $3\,s$

C. $4\,s$

D. $5\,s$

33. With what velocity should a stone be thrown vertically upwards to reach the top of a hill 50 m high? [Take $g = 10\,ms^{-2}$]

A. $10\,ms^{-1}$

B. $31.62\,ms^{-1}$

C. $100\,ms^{-1}$

D. $3.33\,ms^{-1}$

34. A truck travelling on level ground is subject to a resisting force (from the brakes and air resistance) of 250 kN for a distance of 5 km. How much kinetic energy does the train lose?

A. 1250 MJ

B. -1250 MJ

C. 1250 J

D. 1250 KJ

35. A body starts from rest and moves with a uniform acceleration of $10\,ms^{-1}$. Calculate the distance it covers in the 4th second.

A. 35 m
B. 80 m
C. 24 m
D. 45 m

36. A stone is dropped from the top of a building and reaches the ground after 2 seconds. Determine the height of the building. [Take $g = 10\,ms^{-2}$]

A. 19 m
B. 18 m
C. 20 m
D. 25 m

37. A load of mass 70kg is placed in a lift. Calculate the reaction between the floor of the lift and the load when the lift moves upwards with an acceleration of $2\,ms^{-2}$.
[Take $g = 9.8\,ms^{-2}$]

A. 726 N
B. 800 N
C. 850 N
D. 826 N

38. A block of mass 3kg rests on a rough horizontal table. If the coefficient of friction between the block and the table is 0.43, determine the minimum horizontal force that will make the block to slide on the table. [Take $g = 10\,ms^{-2}$]

A. 12.9 N
B. 43.0 N
C. 30.0 N
D. 9.8 N

39. Three points A, B and C have spatial coordinates $(-5,-2,3),(-2,5,3)$ and $(4,5,2)$ respectively. Find the scalar product of displacement vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

A. -42.0
B. -14.0
C. -18.0
D. -24.0

40. The mean score of 100 students in a class that took a Statistics test is 72. If the mean of score of the 70 boys in the class is 75, calculate the mean score of the girls in the class.

A. 70
B. 65
C. 60

D. 58

41. In how many ways can the first, second and third positions be won by six athletes in a race of 100 metres?
A. 20
B. 60
C. 120
D. 100

42. A bag contains 5 white, 2 black and 3 green balls. If three (3) balls are drawn at random without replacement, what is the probability that two (2) balls are white and one (1) ball is black?
A. 1/6
B. 2/5
C. 1/5
D. 9/10

43. When a distribution is moderately skewed with one mode, which of the following relation is correct?
A. Mean – Median = 3(Mean-Median)
B. 2Mean = 3mean-mode
C. Mean + Mode = 3(Mean – Median)
D. Mean – Mode = 3(Mean-Median)

44. Which of the measures of central tendency is often affected by extreme values?
A. Mode
B. Mean
C. Data
D. Median

45. The heights, xcm of a sample of 80 female students are summarized by the equation $\sum(x - 160) = 240$. Compute the mean height of a female student.
A. 243
B. 163
C. 136
D. 83

46. A fair coin is tossed 5 times. Find the probability of obtaining at least 4 heads.
A. $\frac{5}{16}$
B. $\frac{1}{16}$
C. $\frac{3}{16}$

D. $\frac{13}{16}$

47. Deduce the mode of 6, 8, 2, 1, 1, 2, 4, 2, 3.
A. 1
B. 2
C. 6
D. 8

48. A population with variance 16 has unknown mean μ. Let $\overline{X}$ be the mean of a random sample of size 100 from this population. If $\overline{X}$ is used to estimate μ, determine the standard error of $\overline{X}$.
A. 16.00
B. 100.00
C. 0.40
D. 0.16

49. What is the mean value of a binomial variate whose number of repeated trials (n) equal to 8 and probability of success (p) equals 0.4?
A. 8
B. 4
C. 3.2
D. 1.92

50. The probability that a patient will die while undergoing a delicate heart operation is 0.175. What is the probability of the patient surviving from the operation?
A. 0.0875
B. 0.325
C. 0.350
D. 0.825

SECTION B: ESSAY QUESTIONS

Answer FOUR Questions in all; One Question from each Course

MAT 001: PURE MATHEMATICS

1. (a) In a class of 100 students, 35 offer History, 43 offer Geography and 50 offer Economics. 14 students offer History and Geography, 13 offer Geography and Economics and 11 offer History and Economics. The number of students that offer none of the subjects is four times the number of those that offer three subjects.

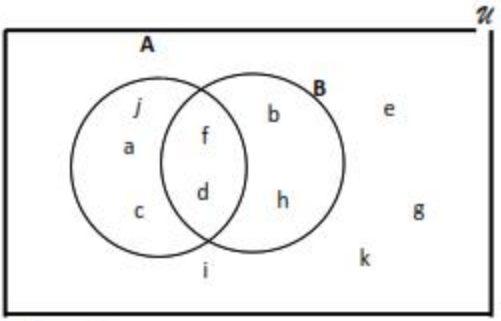
(i) How many students offer three subjects? [4 Marks]

(ii) How many students offer none? [1 Marks]

(iii) How many students offer only one of the subjects? [2 Marks]

(iv) How many students offer History and Geography only? [1 Marks]

(b) Consider the Venn diagram below.



Find the following sets.

(i) $A \cap B^c$. [1 Marks]

(ii) $A^c \cup B$. [1 Marks]

(c) Solve $2\sqrt{x+5} - \sqrt{2x+8} = 2$. [5 Marks]

2.(a) Prove by mathematical induction or otherwise that

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$ [5 Marks]

(b) Let R be the set of real numbers, if binary operation $\otimes$ is defined on $\mathbb{R}$ by $a \otimes b = (a+1)(b+1)$ where a and b belongs to R. Solve the equation $(2 + 4n) \otimes (4 - n) = 5n \otimes n$. [5 Marks]

(c) An A.P. has a common difference of 2 and G.P. has a common ratio of 3. The corresponding terms are added together to produce a new sequence. If the 1st term of the sequence is 8 and its third term is 52, find

(i) the A.P. [2 marks]

(ii) the G.P. [2 marks]

(iii) the new sequence. [1 marks]

MAT 002: CALCULUS

3. (a) A vehicle starts from rests and its velocity is measured every second for 8 seconds as follows:

Time $t(s)$	0	1	2	3	4	5	6	7	8
velocity $v(ms^{-1})$	0	0.4	1.0	1.7	2.9	4.1	6.2	8.0	9.4

The distance travelled in 8 seconds is given by $\int_0^8 v\,dt$.

Estimate this distance using Simpson's rule. [6 Marks]

(b) Evaluate $\int \frac{x+1}{x^2-3x+2}\,dx$ [6 Marks]

(c) Find the differential coefficient of $y = 3x^2 sin\ 2x$ [3 Marks]

4. (a) Obtain the expansion of $e^x sin\ x$ in ascending power of x as far as the term in x^5 [8 Marks]

(b) Evaluate

(i) $\int_0^{\pi/2} cos\,x\,dx$ [3 Marks]

(ii) $\int_0^{\pi/4} (sec^2x + tan^2x)dx$ [4 Marks]

MAT 003: APPLIED MATHEMATICS

5. (a) A block of mass 3.5 kg rests on a rough plane which is inclined at an angle of 42° to the horizontal. If the coefficient of friction between the block and the plane is 0.75, what is the least force P parallel to the plane required to:

(i) start moving the block up the plane? [5 Marks]

(ii) prevent the block from sliding down the plane? (Take $g = 9.8\,ms^{-2}$). [5 Marks]

(b) A body of mass 100 kg is placed in a lift. Find the reaction between the floor of the lift and the body when the lift moves upward with an acceleration of $3.5\,ms^{-2}$ (Take $g = 9.8\,ms^{-2}$). [5 Marks]

6. (a) Three points A, B and C are uniquely orientated in space with coordinates $(\alpha, -1, 2)$, $(5, 2, 4)$ and $(14, \beta, 10)$ respectively. Find the values of α and β such that the three points are collinear. [8 Marks]

(b) A car of mass 160 kg is placed on a rough horizontal road with coefficient of friction 0.5. Find the deriving force when he car moves with

(i) an acceleration of $5\,ms^{-2}$, [4 Marks]

(ii) a uniform speed of $3\,ms^{-1}$. [3 Marks]

MAT 004: STATISTICS

Portions of relevant statistical tables are given behind the questions

7. (a) Suppose the probability function of a discrete random variable X is given by;

$$f(x) = \begin{cases} \dfrac{4!}{(4-x)!\,x!} \cdot \dfrac{1}{16}, & x = 0, 1, 2, 3, 4 \\ 0, & elsewhere \end{cases}$$

(i) find $P(X \leq 3)$, [3 Marks]

(ii) obtain the distribution function of X, [1 Mark]

(iii) find $P(X > 0)$, [2 Marks]

(iv) find the mean and variance of X. [4 Marks]

(b) The following table lists the frequency distribution for 60 rolls of die.

Outcome	1-spot	2-sport	3-sport	4-sport	5-sport	6-sport
Frequency	7	12	8	15	11	7

Test at the 5% significance level whether the null hypothesis that the given die is fair true. [5 Marks]

8. (a)(i) Distinguish between type I and type II errors. [2 Marks]

(ii) Past record shows that the mean score of year II Accountancy students Business Statistics is 56% with a standard deviation of 7%. A random sample of 45 year II students of this department was tested and the mean score was 52%. Verify at α=0.05 if the average score of the students in this course has changed. [9 Marks]

(b) An Accountancy student claims he is 90% sure of the postings he made in all transactions. Would you agree with this claim if in a given transaction, he makes 20 out of 25 postings correctly? Use α=0.05. [4 Marks]

Relevant Statistical Tables

Chi square Table

df α=	0.5	0.250	0.100	0.050	0.025	0.010	0.005
4	3.3567	5.385	7.779	9.488	11.143	13.277	14.860
5	4.351	6.626	9.236	11.070	12.833	15.086	16.750
6	5.348	7.841	10.645	12.592	14.449	16.812	18.548
7	6.346	9.037	12.017	14.067	16.013	18.475	20.278
8	7.344	10.219	13.362	15.507	17.535	20.090	21.955
9	8.343	11.389	14.684	16.919	19.023	21.666	23.589

Standard Normal (Z) Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890

Students' t Distribution Table

	α=	0.25	0.1	0.05	0.025	0.01	0.005
v	1	1.000	3.078	6.314	12.706	31.82	63.66
	2	0.816	1.886	2.920	4.303	6.965	9.925
	3	0.765	1.638	2.353	3.182	4.541	5.841
	4	0.741	1.533	2.132	2.776	3.747	4.604
	5	0.727	1.476	2.015	2.571	3.365	4.032
	6	0.718	1.440	1.943	2.447	3.143	3.707
	7	0.711	1.415	1.895	2.365	2.998	3.499
	8	0.706	1.397	1.860	2.306	2.896	3.355